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MASSILIA.

Elements. By M. G. Rümker.

Computed from the Hamburg Observations, Sept. 30, Oct. 26, and Nov. 19.

M	297° 0′ 46″·0	Nov. 0·0, 1852, Berlin M.T.
ϖ	94 32 39·7	Mean Eq^x. Jan. 0, 1853.
☊	207 8 47·6	
i	0 40 28·1	
φ	10 3 27·5	
Log a	0·3890493	
Log μ	0·2664326	

These elements represent the middle observation, thus:

Computation – Observation $\left\}\begin{array}{l} +0''\cdot 4 \text{ in Longitude} \\ +0\cdot 0 \text{ in Latitude}\end{array}\right.$

An ephemeris based on these elements was sent by M. G. Rümker at the same time: this has been forwarded to our principal observers.

The observations of the solar eclipse of July 28, 1851, which form Part I. of Vol. XXI. of the *Memoirs*, are now published. There are six plates and 120 pages of letterpress. The price to Fellows is 5*s.*, and 10*s.* to the public. There is a separate title-page for the purchasers of the part who may not want the volume.

On a New Method of Computing the Perturbations of Planets. By M. Encke, Director of the Royal Observatory of Berlin. Translated into English and Illustrated with Notes by G. B. Airy, Esq., Astronomer Royal. (Appendix to the *Nautical Almanac* for 1856.)

The main object of the author is to supply an easy and accurate method of computing the perturbations of the various small planets revolving between *Mars* and *Jupiter*, the number of which is constantly increasing from fresh discoveries. The method, however, is applicable to any of the other bodies of the planetary system.

Let t denote the time from which it is proposed to compute the action of the disturbing body. By a slight transformation the differential equations of the motion of the disturbed planet are reduced to the following form:—

$$\xi = \iint \left(\mathrm{X} + \frac{k^2}{r^{\circ 3}} \left(3\, \frac{x^\circ}{r^\circ}\, \delta r - \xi \right) \right) dt^2$$

$$\eta = \iint \left(\mathrm{Y} + \frac{k^2}{r^{\circ 3}} \left(3\, \frac{y^\circ}{r^\circ}\, \delta r - \eta \right) \right) dt^2$$

$$\zeta = \iint \left(\mathrm{Z} + \frac{k^2}{r^{\circ 3}} \left(3\, \frac{z^\circ}{r^\circ}\, \delta r - \zeta \right) \right) dt^2,$$

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x°, y°, z°, being the elliptical co-ordinates of the disturbed planet, and r° its radius vector, as deducible from the elements of the orbit in which it was revolving at the instant t; X, Y, Z, the resolved values of the disturbing force in the directions of corresponding rectangular axes; ξ, η, ζ, the perturbations of x, y, z, counted from t; δr, the perturbation of the radius vector; and k a constant quantity.

These equations are integrated by mechanical quadratures, the successive numerical values of the quantity under the sign of the double integral in each equation being corrected by a peculiar process, founded on the theory of finite differences.

The first step is to divide the period for which the perturbations are to be computed, into a number of sufficiently small intervals of time, each equal to τ, and to calculate the values of X, Y, Z, for the times $t - \frac{1}{2}\tau$, $t + \frac{1}{2}\tau$, $t + \frac{3}{2}\tau$, &c., using for this purpose the elliptical co-ordinates x°, y°, z°, of the disturbed planet. As ξ, η, ζ, $\frac{d\xi}{dt}$, $\frac{d\eta}{dt}$, $\frac{d\zeta}{dt}$, are assumed severally equal to zero at the commencement of the period for which the perturbations are to be computed, the first three values of the quantities under the sign of the double integral (or $\frac{d^2\xi}{dt^2}$, $\frac{d^2\eta}{dt^2}$, $\frac{d^2\zeta}{dt^2}$), are readily found with sufficient accuracy by an approximative process, and from them the corresponding values of ξ, η, ζ, are deduced. For all the subsequent values of ξ, η, ζ, the computation is uniform. The mode of procedure is this:—From the three last values of $\frac{d^2\xi}{dt^2}$ an approximate value of the same quantity for the following argument is readily deduced, and a provisional value of ξ is then found by means of the formula of integration $\xi = {}''f + \frac{1}{12}\cdot\frac{d^2\xi}{dt^2}$, $''f$ being a numerical quantity computed with the utmost facility by the aid of results already obtained. The corresponding values of η and ζ being found by a similar process, the three resulting numbers are employed in computing the second part of the expression under the integral sign in each of the fundamental equations. Finally, the numbers thus found, when applied to X, Y, Z, give the corrected values of $\frac{d^2\xi}{dt^2}$, $\frac{d^2\eta}{dt^2}$, $\frac{d^2\zeta}{dt^2}$, which in their turn lead by double summation to the true values of ξ, η, ζ. By a similar process the values of ξ, η, ζ, for the next argument are ascertained, and so on in succession, the operation being rapidly carried on throughout the entire period for which the perturbations are to be computed.

In order to test the practical utility of this method, M. Encke compared its results with those obtained by the method of the variation of arbitrary constants. For this purpose the perturbations of *Vesta* by *Jupiter* from 1854, May 24, to 1855, October 7, were subjected to a double computation. The following table exhibits the values of the polar co-ordinates of the disturbed planet at the end of this period, as resulting from the use of both methods:—

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	Variation of Constants.	New Method.
	° ′ ″	° ′ ″
Longitude	335 20 6·4	335 20 6·6
Latitude	−5 40 42·3	−5 40 42·3
Log. Rad. Vector	0·3665032	0·3665042

M. Encke mentions as a proof of the easy application of this method, that in computing the perturbations of *Vesta* with given places of *Jupiter* for thirteen epochs, only eleven hours were occupied, notwithstanding that he was frequently interrupted in the course of the operation.

If extreme accuracy is required, the values of x, y, z, resulting from the application of ξ, η, ζ, to x^0, y^0, z^0, may be employed in computing X, Y, Z, and then by repeating the operation we may obtain corrected values of ξ, η, ζ. This process, which gives the perturbations depending on the square of the disturbing force, may be repeated over and over again until the assumed and resulting values of ξ, η, ζ, finally agree. The method, however, is so rapidly exhaustive that this object is generally secured at the first repetition.

When the perturbations are large the method is still practicable, but it becomes necessary in this case to reduce in a corresponding degree the magnitude of the intervals of quadrature.

The following are some of the principal advantages attending this method of computation:—

1. It proceeds from the fundamental equations of motion by a series of arithmetical operations of extreme simplicity, only a few analytical expressions of by no means a complicated character being employed in the computation.

2. It affords the means of immediately correcting the quantities employed in computing the disturbing forces for each successive interval, and thereby leads rapidly to results of great accuracy.

3. Since the perturbations of x, y, z, are of the same order as those of the polar co-ordinates, they serve to afford unequivocal indication of the extent to which the intervals of quadrature ought to be diminished. It is well known that the method of the variation of constants may assign enormous perturbations to the elements while at the same time the actual perturbations may be very insignificant.

The original researches of M. Encke on this subject are contained in Nos. 791, 792, and 814 of the *Astronomische Nachrichten.* It may be mentioned that a similar method had been already proposed by Mr. G. P. Bond, of Cambridge, U.S. (*Memoirs of the American Academy of Arts and Sciences*, vol. iv. part i. p. 189). M. Encke, however, did not become acquainted with this circumstance until he had completed his investigation.